Syllabus

Analysis Qualifying Exam

These are the topics to be covered for preparation for the Qualifying Exam portion in Analysis. It is generally expected that Instructors in Math 703-704 will cover the majority of these topics in some detail.

Metric Spaces

- metric spaces, continuous functions
- separability, completeness
- compactness, Heine-Borel
- connectedness

Complex Analysis

- analytic functions: complex derivatives and Cauchy-Riemann equations, analyticity
- special functions: $\log(z)$, e^z, trig functions
- line integrals, Cauchy's theorem and its consequences: Cauchy integral formula, maximum modulus, power series, Fundamental Theorem of Algebra
- classification of zeros and singularities, Laurent series
- residue theorem, evaluation of integrals and series

Lebesgue theory of Measure and Integration

- outer measure, measurable sets, measure spaces, complete and regular measures
- integration, Fatou's lemma and convergence theorems
- extension theorem, product measure and Fubini's theorem, Lebesgue-Stieltjes integral
- absolute continuity, Vitali's lemma, differentiation theory for monotone functions and integrals, functions of bounded variation
- Egorov and Lusin Theorems
- definition of L^p, Hölder and Minkowski inequalities, completeness of L^p, approximation by step and continuous functions

Reference Texts (listed alphabetically by author):

* L. Ahlfors, *Complex Analysis* (3rd edition), McGraw-Hill, New York, 1978.
* J. Conway, *Functions of One Complex Variable*, Springer-Verlag, New York, 1978.
* G. Folland, *Real analysis. Modern techniques and their applications*, John Wiley&Sons, NY, 1999.
* H. Royden, *Real Analysis* (4th edition), Macmillan Co., New York, 1988.
* W. Ruckle, *Modern Analysis*, PSW-Kent, Boston, 1991.
* W. Rudin, *Real and Complex Analysis*, McGraw-Hill, New York, 1987.
* Saks and Zygmund, *Analytic Functions*, Elsevier, New York, 1971.
* Elias M. Stein & Rami Shakarchi, *Real Analysis*, Princeton Lectures in Analysis, 2005.
* Terence Tao, *An introduction to measure theory*, American Mathematical Society, Providence, RI, 2011.

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